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The Research of the Generalized Fibonacci Sequence -based Propagation

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Abstract

This paper assumed a kind of the propagated phenomenon of flow and load capacity, and tried to find a critical value through the generalized Fibonacci series-based and numerical simulation, which can explain when this node may lead to system breakdown.

Keywords: generalized Fibonacci series-based, avalanche, propagation

1. Introduction

Fibonacci sequence is a very beautiful and harmonious series, and its shape can be explained by a series of square arranged in spiral. The side length of starting square is 1, and the side length of the square on its left is 1. Put another square on the two squares , and its side length is 2, then ,add the squares one by one, which side length is 3、5、8、13、21 and so on. Every one of these numbers equals the sum of the former two numbers, and they just form the Fibonacci sequence. The inventor of Fibonacci sequence was Italian mathematician Leonardo Fibonacci.

On the network there are many similarities between the communication behavior of a phenomenon that is the successive failures on the network. In many networks, one or a few nodes or edges failure (this failure may be random, and may also be caused by the deliberate attack) will be coupled through the relationship between nodes caused by failure of other nodes, so that causing a chain reaction, eventually leading to a considerable number of nodes or even the collapse of the network. This phenomenon is called "Avalanche."

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However, in reality there is a class of the phenomenon of the propagation. In the process of the propagation, when one node with load capacity is watched at one moment, then the flow of that node is related to the flow of the first two moments. This paper assumes a common phenomenon of the nature, that is Fibonacci sequence, and uses the mathematical derivation and numerical simulation to get whether the flow of the node **i**s influenced by a critical value and when the node happened the avalanche, which may lead to the system breakdown.

2. Fibonacci sequence

The famous Fibonacci sequence such as 1，1,2,3,5,8,13,21,34,55…with the following recurrence relations [1]

$$\begin{cases} F_1=F_2=1, F_n=F_{n-1}+F_{n-2}, n>=3; \\ F_{m+n}=F_{n-1}F_m+F_nF_{m+1}, \text{m and n are natural numbers} \end{cases} \quad (1)$$

3.Generalized Fibonacci sequence

Fibonacci sequence has many promotional. A class of generalized Fibonacci sequence is given to discuss whether the process of the spread is influenced by a critical value.

If A_1、A_2、F_1、F_2 are all positive real numbers, This paper discusses the recursive formula, as follows:

$$\begin{cases} F_1=1, F_2=1 \\ F_n=A_1F_{n-1}+A_2F_{n-2}, n=3,4\ldots \end{cases} \quad (2)$$

4. The derivation of the generalized Fibonacci sequence -based

Each node in the network is usually given a certain initial load and capacity. When a node for some reason the load beyond its capacity to produce failure, put the load on the node according to a certain percentage (with the previous time the load on the node) allocated to the next moment that point, we can see sometimes the point will be to rapid growth, which could produce an avalanche phenomenon; but sometimes only to grow to basically stop growing after a certain value, of course, produce a very small likelihood of an avalanche phenomenon.

There is a class of the propagation problem. Every node has load capability. Set the volume of the node i on the moment of t as $F_i(t)$, then the volume of the first two moment $F_i(t-1)$ and $F_i(t-2)$ have certain relation, which is assumed as ω.Any impact on the type of the communication process? If so, how they affect the network nodes with a capacity of spread? The paper has researched the process of the propagation which is based upon the generalized Fibonacci mode.

$$\begin{aligned} F_i(t) &= F_i(t-1)+\omega[F_i(t-1)-F_i(t-2)] \\ &= [1+\omega]F_i(t-1)-\omega F_i(t-2) \\ &\text{and } \omega>0 \end{aligned} \quad (3)$$

So the formula（3）is according with the generalized Fibonacci sequence formula（2）.

$$\begin{cases} A_1=1+\omega \\ A_2=-\omega \end{cases}$$

If $F_n=F_i(t)$, $F_{n-1}=F_i(t-1)$, $F_{n-2}=F_i(t-2)$

According to the formula（3）.

$$\begin{cases} F_n=(1+\omega)F_{n-1}-\omega F_{n-2} \\ F_0=0, F_1=1, F_2=1+\omega \end{cases} \quad (4)$$

If an= Fn an-1= Fn-1 an-2= Fn-2

$$an=(1+\omega)an-1-\omega an-2 \tag{5}$$

If an= a2 , an-1= a1 , an-2= a0,among that a is the constant to be determined, call it the test solution or special solution, substituted into the recursive formula, so：

a2=(1+ω)a-ω

a2-(1+ω)a+ω=0

so

$$a = \frac{1+\omega \pm |1-\omega|}{2} \tag{6}$$

So have two solutions, such as:

$$\begin{cases} Fn[1]=a1n=1n \\ Fn[2]=a2n=\omega n \end{cases} \tag{7}$$

It can be proved that a combination of these two solutions (K1, K2 are arbitrary constants)

$$Fn=K1Fn[1]+K2Fn[2]=K1+K2\omega n \tag{8}$$

Still in line with the solution of the recursive formula, call it “general solution”.

Solve the K1,K2 in the Fn, that make F1=1,F0=0.

$$\begin{cases} K1+K2\omega 0=0 \\ K1+K2\omega 1=1 \end{cases} \tag{9}$$

So,

$$\begin{cases} K1=1/(1-\omega) \\ K2=1/(\omega-1) \end{cases} \tag{10}$$

Then the general formula of the Fibonacci sequence is derived by the formula (8)

$$F_n = \frac{1-\omega^n}{1-\omega} \tag{11}$$

To derive f (t) is a function about time.

$$f(t) = \frac{1-\omega^t}{1-\omega} \tag{12}$$

Powdiff $f(t)$

$$\ln f(t) = -t\frac{\ln\omega}{1-\omega}$$

$$\frac{f'(t)}{f(t)} = -\frac{\ln\omega}{1-\omega}$$

$$f'(t) = -\frac{\ln\omega}{(1-\omega)^2(1-\omega^t)} > 0 \tag{13}$$

So $f'(t)$ is the exponential function of time t.

If ω>1, $f(t)$ is rapid growth in avalanche, then if 0<ω<1, $f(t)$ is the slow growth to close a value.

5. Generalized Fibonacci sequence derivation MATLAB simulation curve

We use the tool of the MATLAB to validate and simulate the hypothesis by using the formula

$$f(t) = (1 - \omega^{t})/(1 - \omega) \qquad (12)$$

the results are as follows:

When ω=0.9, as can be seen from the following figure that f(t) is close to 10 with the slow growth.

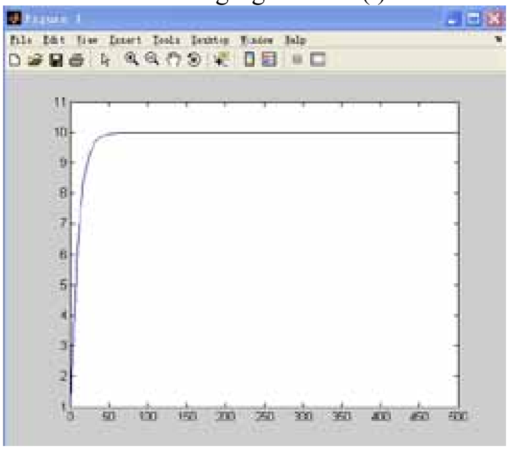

Figure (1): when ω = 0.9, the display of the function changes with time t

When ω =1.1, as can be seen from the following figure that f (t) showed rapid growth in avalanche.

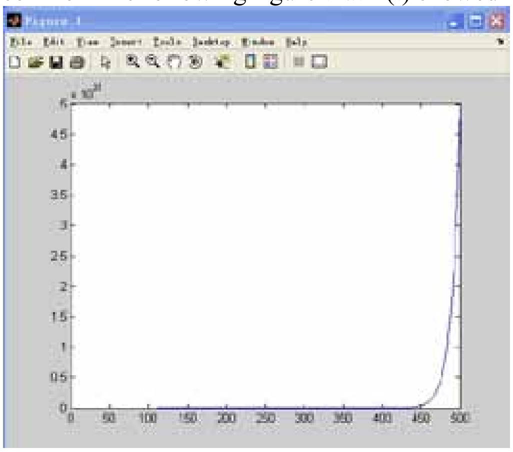

Figure (2): when ω = 1.1, the display of the function changes with time t

6. Conclusion

The paper has used the Mathematical derivation and numerical simulation based on the generalized Fibonacci sequence to find a critical value, which is used to explain in what condition the avalanche happened to the node with the probability of system breakdown.The avalanche will happened in exponential order to the node with the probability of system breakdown when ω, the value of key nodes observed during the transmition,is bigger than 1 and the whole system will be stable when ω is between 0 and 1.

Through generalized Fibonacci series mathematical derivation and numerical simulation, we find that electricity network and the Internet and so on, which is the load capacity of the network. we study the network, a point change over time, changes in the capacity of the point whether the degree of rapid growth? Or stable in the vicinity of a value? Only in reality, it is very important, but how to find and control the critical value of ω is more important, we must try to control the ω range between 0 and 1, so as to effectively control the disastrous avalanche incidents.

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